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Group problems for 9/8/17

R3R.1

In 2095, a message arrives at earth from the growing colony at Tau Ceti (11.9 y from earth). The message asks for help in combating a virus that is making people seriously ill (the message includes a complete description of the viral genome). Using advanced technology available on earth, scientists are quickly able to construct a drug that prevents the virus from reproducing. You have to decide how much of the drug can be sent to Tau Ceti. The space probes available on short notice could either boost 200 g of the drug (in a standard enclosure) to a speed of 0.95, 1 kg to a speed of 0.90, 5 kg to a speed of 0.80, or 20 kg to a speed of 0.60 relative to the earth. The only problem is that a sample of the drug in a standard enclosure at rest in the laboratory is observed to degrade due to internal chemical processes at a rate that will make it useless after 5.0 y.

1. Explain why it is possible to send the drug to Tau Ceti, even though the ship must travel for more than 11.9 y.

Let event A be the probe launch from earth and event B be the probe arrival at Tau Ceti. Because the probe moves at a constant speed, it is inertial, so a clock on the probe will measure the spacetime interval, . By the metric equation,. According to the metric equation, the spacetime interval will always be less than the coordinate time . We can calculate the coordinate time using , so for a very large , will be very small, but will be even smaller coordinate that is, the coordinate time of the time on board the probe will be much less than the coordinate time between the launch of the probe and its arrival at Tau Ceti as measured in the reference frame of the earth.

1. How much can you send?

By the metric equation,

Our different values for are

so the only probe that will make it with any of the drug intact will be the probe sent at a speed of .95 carrying 200g of drug.

R4A.3

Consider the Global Positioning System satellites described in problem R4M.6. Again, for simplicity’s sake, suppose the earth is not rotating. Let be the time between two events that bracket one complete satellite orbit as measured by a clock at rest with respect to the earth but so far away that the effect of the earth’s gravity on its rate is negligible. (We’ll call this “the clock at infinity.”)

1. The satellite’s speed is *,* where *G* is the universal gravitational constant (2.475 × 10–36 s/kg in SR units), *M* is the earth’s mass, and *R* = 26,600 km is the orbit’s radius. Assuming that << 1, use the binomial approximation to find an expression (in terms of *G*, *M*, and *R*) for the discrepancy between what the clock at infinity and the satellite’s clock measure for a full orbit due to the satellite’s motion.
2. General relativity states that a clock at rest a distance *R* from the center of a planet of mass *M* runs more slowly than the clock at infinity by the factor (in SR units) due to the planet’s gravitational field. Find a symbolic expression for the gravity-induced discrepancy between what the clock at infinity and the stationary clock at *R* measure for a full orbit.
3. Similarly find the gravity-induced discrepancy between what the clock at infinity and a clock on the earth’s surface at radius *Re* measure for a full orbit.
4. Find a symbolic expression for the *total* discrepancy between what the satellite’s clock and the clock on the earth’s surface measure for a full orbit, taking into account all of these effects.

**(e)** Evaluate numerically and interpret its sign.

R5R.1

**R5R.1** Each Global Positioning System (GPS) satellite constantly broadcasts a signal that specifies what time the signal is sent (according to an atomic clock on the satellite) as well as information about that satellite’s location when the signal is sent. A GPS receiver uses the information sent from multiple satellites to find its location by doing a complicated calculation that accounts for the signal travel time from the satellites, the rotation of the earth, and a variety of effects predicted by both special and general relativity.

To see just some of the effects of special relativity that are involved, consider a starkly simplified GPS system where the satellites fly at a constant speed of *β* a negligible distance above the *x* axis on a flat earth. Assume the satellites’ atomic clocks are synchronized in the satellites’ frame, and that you are standing somewhere along the *x* axis. At a certain instant, your GPS receiver simultaneously receives a signal from somewhere along the -*x* direction relative to you from a satellite *A* and another signal from somewhere along the +*x* direction from a satellite *B*. The signals both state the same signal departure time (= = 0 in the satellite’s frame) and specify that the satellites are at locations and , respectively, at that time.

1. Assume that the Galilean transformation equations are true and that both signals have the same speed in the earth frame. Show that your position is

Using the Galilean transformation equations,

At time , , , so

1. Assume that and let and (to pick something arbitrary). Carefully construct a spacetime diagram of the situation, and argue from the diagram that

As we can see from the two observer spacetime diagram, the time that the satellites send their signals are the same and their point of intersection on the -axis shows that my position is, and seeing that ,

1. Using this diagram to guide your thinking, develop a general equation that calculates your position along the *x* axis in the flat earth’s frame in terms of *β* and the reported values of and , taking account of special relativity. Check that your proposed equation yields results consistent with your diagram for specific values of , , and given in part (b). (*Hint:* Let event *C* be the event of your receiver obtaining both signals. Can you find the coordinates of event *C* in either the spaceship frame or the earth frame? If the former, you can transform to get the coordinates in the earth frame.)

The general equation of the position is given by the Lorentz transform:

In order to express and in terms of and , we need to find equations for the satellite signals in the reference frame of the satellites. Using point-slope form,

and . The place where these lines intersect is the coordinate of event C. Solving for yields

Solving for yields

Now we plug these into our Lorentz transform:

Using the info from part b,

This value agrees with the value given by our graph, so our equation is correct.

1. Suppose the satellites’ common speed is 3.9 km/s (which is roughly the real GPS satellites’ orbital speed) and that their reported positions are = -3000 km and = +9000 km. By how much would the naive calculation of part (a) be in error, according to your relativistic formula? Is this significant?

By our relaticistic formula,

But by the Galilean based formula,

So the error between the relativistic value and the Galilean value is

This is a very small value, and I am surprised that the Galilean transformation equations was so accurate.